6.2 Apply Properties of Rational Exponents Key

The properties of integer exponents you learned in 5.1 can also be applied to rational exponents. See the table on page 420 for a complete list.

<u>Example 1</u>
Use the properties of rational exponents to simplify the expression.

a.) $\left(5^{1/3} \cdot 7^{1/4}\right)^3$

$\boxed{5 \cdot 7^{\frac{3}{4}}}$

b.) $2^{3/4} \cdot 2^{1/2}$

$2^{3/4} \cdot 2^{2/4}$

$\boxed{2^{\frac{5}{4}}}$

c.) $\dfrac{3}{3^{1/4}}$

3^1

$1 - \frac{1}{4}$

$\frac{4}{4} - \frac{1}{4} = \frac{3}{4}$

$\boxed{3^{\frac{3}{4}}}$

d.) $\left(\dfrac{20^{1/2}}{5^{1/2}}\right)^3$

$\left(\dfrac{\sqrt{20}}{\sqrt{5}}\right)^3 = \left(\sqrt{4}\right)^3$

$= 2^3$

$= 8$

Properties of Radicals

Product property of Radicals

$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Quotient Property of Radicals

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

<u>Example 2</u>
Use the properties of radicals to simplify the expression.

a.) $\sqrt[3]{125} \cdot \sqrt[3]{8}$

$5 \cdot 2$

$\boxed{10}$

b.) $\dfrac{\sqrt[5]{96}}{\sqrt[5]{3}} = \sqrt[5]{\dfrac{96}{3}}$

$\sqrt[5]{32}$

$\boxed{2}$

c.) $\sqrt[4]{27} \cdot \sqrt[4]{3}$

$\sqrt[4]{81}$

$\boxed{3}$

A radical with index n is in **simplest form** if the radicand has no perfect nth powers as factors and any denominator has been rationalized.

Ex: $\sqrt{48}$

$\sqrt{16} \cdot \sqrt{3}$

$4\sqrt{3}$

*find perfect square

<u>Example 3</u>
Write the expression in simplest form.

find perfect cube

a.) $\sqrt[3]{104}$

$\sqrt[3]{8} \cdot \sqrt[3]{13}$

$\boxed{2\sqrt[3]{13}}$

$\sqrt{27}$

$3 \cdot 3 \cdot 3$

need 1 more 3

b.) $\dfrac{\sqrt[4]{10}}{\sqrt[4]{27}} \cdot \dfrac{\sqrt[4]{3}}{\sqrt[4]{3}}$

$\dfrac{\sqrt[4]{30}}{\sqrt[4]{81}} = \boxed{\dfrac{\sqrt[4]{30}}{3}}$

c.) $\sqrt[5]{\dfrac{3}{4}} = \dfrac{\sqrt[5]{3}}{\sqrt[5]{4}} \cdot \dfrac{\sqrt[5]{8}}{\sqrt[5]{8}}$

$\sqrt{4}$

$2 \cdot 2$

*need 3 more 2's → 8

$= \dfrac{\sqrt[5]{24}}{\sqrt[5]{32}} = \boxed{\dfrac{\sqrt[5]{24}}{2}}$

Radical expressions with the same index and radicand are **like radicals**. To add or subtract like radicals, use the distributive property. $3\sqrt{2}+5\sqrt{2}=8\sqrt{2}$

Example 4
Simplify the expression.

a.) $7\sqrt[5]{12}-\sqrt[5]{12}$

$\boxed{6\sqrt[5]{12}}$

b.) $4\left(9^{2/3}\right)+8\left(9^{2/3}\right)$

$12\left(9^{2/3}\right)$

c.) $\sqrt[3]{5}+\sqrt[3]{40}$

$\sqrt[3]{5}+\sqrt[3]{8}\sqrt[3]{5}$

$1\sqrt[3]{5}+2\sqrt[3]{5}$

$\boxed{3\sqrt[3]{5}}$

d.) $\sqrt[3]{81}-\sqrt[3]{24}$

$\sqrt[3]{27}\sqrt[3]{3}-\sqrt[3]{8}\sqrt[3]{3}$

$3\sqrt[3]{3}-2\sqrt[3]{3}$

$\boxed{1\sqrt[3]{3}}$

Example 5
Simplify the expression. Assume all variables are positive.

$\frac{5}{1}\cdot\frac{1}{5}=1$ $\frac{30}{1}\cdot\frac{1}{5}=6$

a.) $\sqrt[4]{625z^{12}}$

$\sqrt[4]{625}\sqrt[4]{z^{12}}$

$\boxed{5z^3}$

$z^3\cdot z^3\cdot z^3\cdot z^3=z^{12}$

b.) $\left(32m^5n^{30}\right)^{1/5}$

$\sqrt[5]{32}\,mn^6$

$\boxed{2mn^6}$

c.) $\dfrac{56ab^{3/4}}{7a^{5/6}c^{-3}}$

$\boxed{8a^{1/6}b^{3/4}c^3}$

$\frac{1}{1}-\frac{5}{6}$

$\frac{6}{6}-\frac{5}{6}=\frac{1}{6}$

d.) $\sqrt[3]{6x^4y^9z^{14}}$

$x\cdot x\cdot x\;x$ $\quad y\cdot y\cdot y\;y\cdot y\cdot y\;y\cdot y\cdot y=y^3$

$\sqrt[3]{6}\sqrt[3]{x^4}\sqrt[3]{y^9}\sqrt[3]{z^{14}}$

$\boxed{xy^3z^4\sqrt[3]{6xz^2}}$

e.) $\sqrt[7]{\dfrac{p^8}{q^5}}$

$\dfrac{\sqrt[7]{p^8}}{\sqrt[7]{q^5}}\cdot\dfrac{\sqrt[7]{q^2}}{\sqrt[7]{q^2}}=\dfrac{\sqrt[7]{p^8q^2}}{\sqrt[7]{q^7}}$

$=\boxed{\dfrac{p\sqrt[7]{pq}}{q}}$

Example 6
Perform the indicated operation. Assume all variables are positive.

a.) $18\sqrt[3]{u}-11\sqrt[3]{u}$

$\boxed{7\sqrt[3]{u}}$

b.) $15a^4b^{2/3}-8a^4b^{2/3}$

$\boxed{7a^4b^{2/3}}$

c.) $10\sqrt[4]{5s^7}-s\sqrt[4]{80s^3}$

$10\sqrt[4]{5s^7}-s\sqrt[4]{16}\cdot\sqrt[4]{5s^3}$

$10s\sqrt[4]{5s^3}-2s\sqrt[4]{5s^3}$

$\boxed{8s\sqrt[4]{5s^3}}$